

DESIGN RESEARCH: DEVELOPMENT OF LEARNING TRAJECTORIES IN REALISTIC MATHEMATICS EDUCATION ON RELATION AND FUNCTION MATERIALS

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Abstract. The purpose of this study is to produce a learning trajectory on the material of relations and functions in class VIII. This type of research is Design Research proposed by Gravemeijer & Cobb, carried out through 3 stages, namely the research preparation stage, research in the classroom, and retrospective analysis. The research was conducted at SMP Negeri 4 Mori Utara with the research subjects totaling 12 students and consisting of three meetings, namely 1) the first meeting: Relation Concept, Relationship Presentation and Domain Concept, Codomain and Range, 2) second meeting: Function Concept, Function Characteristics and Correspondence One-on-one and 3) third meeting: Notation and Function Value. The research instruments used are in the form of learning trajectory hypotheses and data collected through observations, sound recordings, and Student Activity Sheets. The results showed that the learning trajectory in the Relation and Function material with a RME approach designed to improve mathematical problem-solving ability can be applied in the learning process. The resulting learning path is in the form of a flow of activities and activities carried out by students in class in order to achieve the learning goals that have been previously set. Meanwhile, this study was conducted only to see the application of the hypothesis of the learning trajectory of the Relation and Function material in the RME approach.

Keywords: Learning Trajectory, RME, Problem Solving Skills

Abstrak. Tujuan penelitian ini adalah untuk menghasilkan lintasan belajar pada materi relasi dan fungsi di kelas VIII. Jenis penelitian ini adalah Design Research yang dikemukakan oleh Gravemeijer & Cobb, dilaksanakan melalui 3 tahapan, yaitu tahap persiapan penelitian, penelitian di dalam kelas, dan analisis retrospektif. Adapun penelitian dilakukan di SMP Negeri 4 Mori Utara dengan subjek penelitian berjumlah 12 orang siswa dan terdiri dari tiga pertemuan, yaitu 1) pertemuan pertama : Konsep Relasi, Penyajian Relasi dan Konsep Domain, Kodomain dan Range, 2) pertemuan kedua : Konsep Fungsi, Ciri-ciri Fungsi dan Korespondensi Satu-satu dan 3) pertemuan ketiga : Notasi dan Nilai Fungsi. Instrumen penelitian yang digunakan berupa hipotesis lintasan belajar dan data dikumpulkan melalui observasi, rekaman suara, dan Lembar Aktivitas Siswa. Hasil penelitian menunjukkan bahwa lintasan belajar pada materi Relasi dan Fungsi dengan pendekatan PMR yang dirancang untuk meningkatkan kemampuan pemecahan masalah matematis dapat diterapkan dalam proses pembelajaran. Lintasan belajar yang dihasilkan berupa alur kegiatan serta aktivitas yang dilakukan siswa di kelas guna mencapai tujuan pembelajaran yang sudah ditetapkan sebelumnya. Adapun, penelitian ini dilakukan hanya untuk melihat penerapan hipotesis lintasan belajar materi Relasi dan Fungsi pada pendekatan PMR.

Kata Kunci: Lintasan Belajar, PMR, Kemampuan Pemecahan Masalah

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INTRODUCTION

Education is one of the things that Indonesia is still fighting for today. In the Preamble to the 1945 Constitution, the 4th paragraph clearly states that educating the nation's life through education is one of Indonesia's national goals. This is certainly a big picture of the hope of the Indonesian nation in improving the quality of education throughout Indonesia so that it can compete abroad. Therefore, to realize this goal, the education system in Indonesia sets a process standard in the implementation of learning in the hope of creating active and innovative learning and providing opportunities for students to be able to develop insights, attitudes and creative power in accordance with their interests and talents so that they can achieve graduate competencies (Permendikbud, 2016). This also encourages all education parties to be proactive in building a better quality of education in Indonesia.

In Indonesia, education is one of the efforts made by the government to be able to create a life for developed people and be able to compete in the future. With education, an independent national character will be formed, and has skills in every area of life. However, in line with efforts to improve education in Indonesia, the learning process that occurs in schools does not run as expected, including mathematics learning. Mathematics as a subject that must be pursued by students until now is still seen as difficult to understand (Ulya & Agustyarini, 2020). The abstractness reflected by mathematics makes many students avoid even closing themselves to mathematics. This is certainly the biggest scourge in the world of education which not only interferes with the running of the learning process but also affects the achievement of the quality of education produced.

Apart from the various methods, approaches and learning strategies used, there are still many problems that are often found in the field. In mathematics learning itself, many students who have difficulties and even have very low mathematical abilities. Based on the results of preliminary studies, mathematical ability, especially in the problem-solving ability of students at SMP Negeri 4 Mori Utara, is very low. This is found based on the results of working on midterm test questions which include mathematical problem-solving indicators. The difficulties experienced by learners are found when analyzing each known component in the question. This is due to the lack of understanding of mathematical concepts as a result of conventional learning strategies applied by teachers.

Mathematical problem solving is an activity or thinking activity in describing knowledge related to the process of solving mathematical problems systematically (Kai Kow Joseph Yeo, 2009). Mathematical problem solving is an activity that encourages students to think multidimensionally in looking for possible answers to a problem so that they can build better

observation and analysis skills in solving (Rahmani & Widyasari, 2018). Branca (1980), interprets mathematical problem solving into 3 parts, namely, 1) as a goal and reason for mathematics learning, 2) as a process and part of the strategy that is passed through and 3) as a basic skill that students must have.

Polya (1973) put forward 4 indicators in problem solving, namely 1) Understanding problems, namely the process of identifying and mastering information that is already or is not yet known, 2) Planning for completion, namely the process of preparing patterns that will be used in solving problems, 3) Solving problems according to plan, namely the process of implementing solutions, and 4) Re-testing and concluding solutions, namely the activity of reviewing the process and results whether they are in accordance with those Planned. Problem-solving ability is defined as cognitive skills that must be possessed by students so that self-habituating is needed in solving mathematical problems through problems in everyday life. Therefore, students are expected to be able to understand, strategize, and solve problems systematically according to their stages so as to provide a meaningful mathematics learning experience.

In line with the Kemendikbud (2014) which states that the purpose of doing mathematics learning to students is to have the ability to: 1) Understand and apply mathematical concepts, 2) Reason to explain the nature and ideas of mathematical ideas, 3) Solve problems that include understanding, designing, and solving mathematical models, 4) Communicating the idea of mathematics into other representations and 5) Being positive as a form of appreciation for mathematics in everyday life. So based on the purpose of mathematics, every mathematics learning should begin with a real experience of mathematical concepts which involve solving students' problems with mathematical processes. Through contextual learning, learners will be directly involved in the discovery of concepts and can activate their reasoning intuition when encountering problems in real life so that it can make it easier for students to understand and even solve the problems they face.

Therefore, in overcoming these problems, a certain learning approach is needed that is able to improve the ability to understand concepts and solve mathematical problems of students, namely by using the Realistic Mathematics Education learning approach. Realistic mathematics Education is a learning approach that is only devoted to mathematics learning. Specifically, this approach was first developed in 1970 in the Netherlands and focuses on learning the discovery of mathematical ideas that involve activities in everyday real life (Tambunan, Sitinjak, & Tamba, 2019). The formation of the Realistic Mathematics Education approach is based on the idea that mathematics is part of human values so that it

must be linked to real-world life (Freudenthal, 2002). Realistic Mathematics Education is an approach centered on the concrete world with the aim of helping students understand mathematical concepts, improve mathematical abilities, and find mathematical ideas through the process of analyzing student problems in mathematics learning (Lestary, Ahmad, & Lubis, 2020).

The process of real-world exploration of Realistic Mathematics Education is applied through problems experienced by students themselves. In line with Yurnalis & Rahmi (2017) which states that the meaningfulness of Realistic Mathematics Education arises if the learning process is associated with problems that can be imagined or felt in real terms by students, because realistic mathematics education is not only a learning resource but is a process of learning activity. For this reason, the use of a good problem context in Realistic Mathematics Education is expected to make a major contribution in building mathematical concepts and improving students' mathematical abilities.

Research by Harahap & Lubis (2019) shows that Realistic Mathematics Education is effectively used in improving students' mathematical problem-solving ability. Realistic Mathematics Education involves student activities in the rediscovery of mathematical concepts where students have the opportunity to get mathematical concepts through learning that uses existing contextual problems (Fitri, 2016). This learning approach provides a space for students to find understanding of mathematics through modeling activities to solve problems through teacher guidance. Therefore, the amount of contribution made by students in this learning determines the success of the application of the Realistic Mathematics Education learning approach.

In addition, in Realistic Mathematics Learning, a learning trajectory is needed as a learning design that regulates the entire flow of learning activities. Learning path is a series of learning paths that contain the alleged possibility of student activities to improve their thinking skills in accordance with the goals that have been set (Wandanu, Mujib, & Firmansyah, 2020). Realistic Mathematics Education is an approach that allows the development of learning trajectory hypotheses as an illustration of the learning step process that is arranged systematically so that it is expected to help students experience meaningful learning. In line with Liu (2020) who states that the learning trajectory hypothesis becomes an instrument that can connect learning activities and mathematical theories. For this reason, in planning mathematics learning, teachers must be able to consider every aspect and conjecture of possibilities that can affect mathematics learning.

Relationships and functions are one of the mathematics materials of class VIII that can use contextual problems in the learning process. Relationship is defined as a relationship and this has often been encountered and even felt directly by students in everyday life. Examples of relationships in contextual problems include family members, passions, ownership of goods, and others. Given that in Realistic Mathematics Learning, the starting point of learning begins by using the context of concrete objects in finding and building mathematical concepts (Ningsih, 2014), then in the material Relationships and Functions, the context of real problems is used with the help of "RESI" in Realistic Mathematics Learning. The use of contextual images in "RESI" is made simply, so that it can help students more easily understand and find the concept of relationships and functions. The relationship between the material and "RESI" can be found in the use of the media which in its application gives a clear picture of a relationship or relationship formed by two sets. Thus, the context of real problems assisted by the learning tool "RESI" can be used as a learning resource by students in learning the concepts of relationships and functions.



Figure 1. "RESI"

On the basis of the above problems, the researcher intends to conduct a development research entitled "Design Research: Development of Learning Trajectories in Realistic Mathematics Education to Improve Mathematical Problem Solving Ability". This research was carried out with the aim of seeing how the activity of applying the Learning Trajectory Hypothesis of the Realistic Mathematics Investigation approach to the learning process and its effectiveness in improving students' problem-solving abilities. The Realistic Mathematics Education Approach is applied to the material Relationships and Functions of class VIII junior high school. This research is carried out with the hope of being able to become a solution to problems in the learning process, especially improving students' mathematical problem-solving skills in the relationship and function material.

METHOD

This research is a Design Research research, which is research that focuses on the development of theory and learning activities (Gravemeijer, 2006). This research was carried out in accordance with the 3 stages proposed by Gravemeijer & Cobb (2006), namely preparation, research in the classroom and retrospective analysis. The study was conducted at SMP Negeri 4 Mori Utara, North Morowali Regency for the 2021/2022 school year from November to December 2021 with the following stages described.

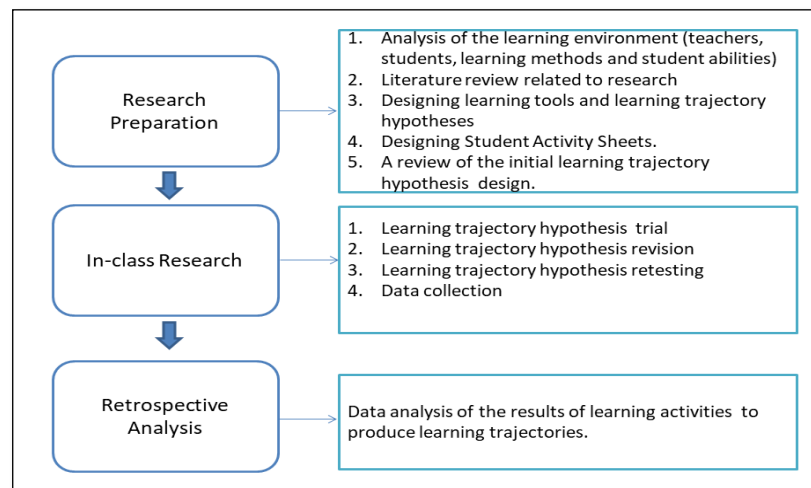


Figure 2. Research Stages

The subjects of this study were 12 students of class VIII of SMP Negeri 4 Mori Utara. The instrument used is in the form of a pre-designed learning trajectory hypothesis. Data was collected through observation, voice recorders and Student Activity Sheets that have been designed to improve students' problem-solving abilities. Furthermore, the data are analyzed descriptively based on the results of observations in the learning process. The analysis process is carried out by comparing the hypothesis of learning trajectory and the process of learning activities that take place in the classroom. All data collected through observation and voice recorders are written into the form of transcripts to be able to provide an overall picture of learning activities in the classroom. Then, based on the results of data analysis, conclusions were drawn on the research that had been carried out.

RESULTS

The results of the study explained in detail how the learning trajectory that occurred in the stages of learning realistic mathematics on the material of relationships and functions at three meetings, namely as follows.

Table 1. Learning Meetings

Meeting	Material	Learning Activities
Meeting 1	The concept of relationships	Expressing the relationship of an object in the surrounding environment Understanding the concept of relationships in everyday life Expressing relationships in the form of arrow charts, cartesian charts, and sequential pairs Represents the domain, codomain, and range of a relationship.
Meeting 2	The concept of function	Declaring a relationship Investigating the traits of the function Investigating the characteristics of one-on-one correspondence
Meeting 3	The notation and value of the function	Solving contextual problems related to function values

The research activities were carried out in two trials, the goal was to be able to find new theories in the learning trajectory that were as expected. The first test was carried out on 8 students, while the second test was carried out on 4 students.

First Trial

In the first trial, it was found that the designed Learning Trajectory Hypothesis has not been able to provide maximum results. At the first meeting, Learning Trajectory Hypothesis design errors were found, especially in the results of Student Activity Sheets 1, namely students had difficulty in inferring the concept of relationships and were wrong in presenting relationships with arrow diagrams and sequential pairs. In this activity, students were asked to write down the grouping of animal types that they knew from science subjects. The material of grouping animal types has actually been studied by students since elementary school and can basically be understood by students. However, the use of this context is considered too difficult to use because it already involves an abstract thought process, as a result of which students have difficulty in understanding the concept of relationships.

In addition, there are design errors in the activity of determining the domain, codomain, and range, where students have difficulty in understanding the concepts of domain, codomain, and range because the selection of examples of relationships formed since the initial activity cannot provide a clear picture of the form of relationships, so this activity requires a little material explanation from the teacher. Another error found was that the

student could not correctly remember the concept of the set. As a result, at the time of writing members of the same set numbering more than one, students write them repeatedly.

KEGIATAN 1: KONSEP RELASI
 Tuliskan 3 pengelompokan jenis hewan yang kalian ketahui!

- Berdasarkan Tempat tinggal, yaitu Air, darat, dan ~~angin~~
- Berdasarkan Cara bergerak, yaitu merayap, terbang, melompat
- Berdasarkan Jenis makanan, yaitu omnivora, karnivora, herbivora

Pilihlah salah satu pengelompokan jenis hewan, setelah itu pilihlah 5-7 hewan berdasarkan gambar hewan yang disediakan.

Pengelompokan jenis hewan : Tempat tinggal
 Hewan yang dipilih : Serigala, ayam, kelinci, ular, kucing, ikan, ikan hiu
 Kelompokkan setiap hewan tersebut ke dalam tabel berikut ini.

Hewan	Tempat tinggal
Seri galo	darat
ayam	darat
kelinci	darat
ular	darat
kucing	darat
ikan	air
ikan hiu	air

Hewan **tempat tinggal**

$R = (\text{Serigala}, \text{darat})$
 $R = (\text{ular}, \text{darat})$
 $R = (\text{ayam}, \text{darat})$
 $R = (\text{kelinci}, \text{darat})$

$R = (\text{ikan}, \text{air})$
 $R = (\text{ikan hiu}, \text{air})$
 $R = (\text{kucing}, \text{darat})$

Figure 3. Errors in the First Trial Student Activity Sheets Work Results

At the second meeting, the activity carried out by the students is to find the characteristics of the function and the one-on-one correspondence based on the examples of relationships they find. In this activity, students are asked to create relationships based on name, date of birth, hobby and favorite subjects, then declare them into an arrow diagram. The error obtained in this case is the lack of understanding of the concept of the set, where the student writes down the members of the same set of more than one. Meanwhile, at the third meeting, namely the activity of solving contextual problems related to function values, no errors were found in both the Learning Trajectory Hypothesis design and the Student Activity Sheets results. Therefore, researchers revised the Learning Trajectory Hypothesis which had been designed based on the results of the first trial and input and suggestions from experts.

Second Trial

Meanwhile, the learning trajectory in the Relation and Function material in the second trial was described as follows.

First Meeting: Relation Concept, Relationship Presentation and Domain Concept, Codomain and Range

The learning process of the first meeting is carried out with the aim that students can understand the concept of relationships, the presentation of relationships and be able to state the domain, codomain and range of a relationship. Learning activities are carried out using the context of examples of problems that occur in the daily environment and are connected with other sciences. As for the implementation of learning activities, "RESI" is used as a tool that bridges students to think from contextual mathematics to abstract mathematics.

Therefore, students are expected to be able to master the concept and presentation of relationships and state the domain, codomain, and range of a relationship.

The learning activities of the first meeting begin with the process of reviewing the set material as a prerequisite material for relationships and functions. In this activity, students are asked to provide examples of sets by mentioning and selecting several images from the "RESI" that have been provided as an example of a set. However, in this activity, students of each student are also asked to provide examples along with how to write the set of objects in the surrounding environment. In accordance with the teacher's guess, students can explain the meaning of the set and be able to give examples of the set of objects that are around them. Next, the teacher distributes Student Activity Sheet 1 and explains the instructions for filling out Student Activity Sheet 1 and the purpose of the activity to be carried out.

- Teacher : “Kalian pasti sudah pernah belajar pengelompokkan jenis hewan berdasarkan jenis makanan di pelajaran IPA, kan?”
- Student : “Sudah bu ”.
- Teacher : “Ada berapa pengelompokkan jenis hewan berdasarkan jenis makanan?”
- Student : “Ada tiga bu... Ada Omnivora, ada Herbivora, ada Karnivora”
- Teacher : “Apa itu herbivora? Coba sebutkan contohnya.”
- Student : “Herbivora itu hewan pemakan tumbuhan.
- Student1 : “Contohnya ada sapi, kambing..”
- Teacher : “Kalau karnivora dan omnivora bagaimana? Apa itu karnivora dan omnivora?”
- Student : “Kalau karnivora itu hewan yang pemakan daging sedangkan omnivora hewan pemakan segalanya..daging dan tumbuhan”
- Teacher : “Contoh karnivora dan omnivora apa?”
- Student2 : “Karnivora itu kayak kucing, singa, harimau. Kalo omnivora itu monyet...ayam juga ”

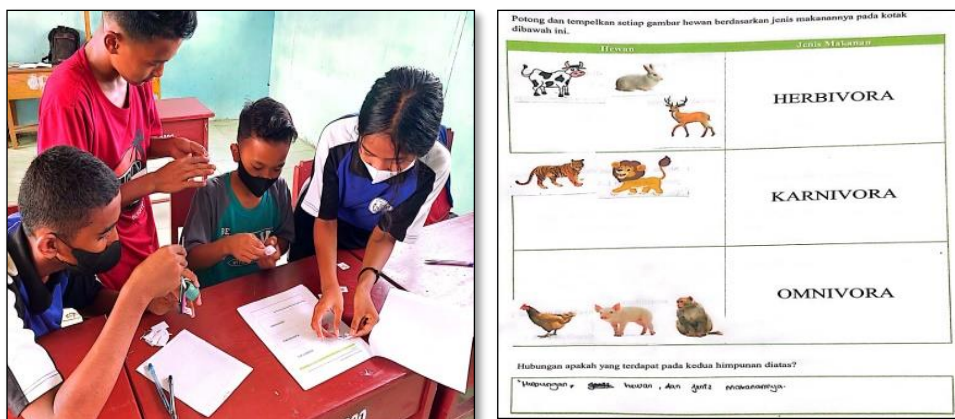


Figure 4. Animal Grouping Activities

Based on the results of the conversation, it is in accordance with the teacher's guess that students can remember and distinguish animals that belong to the herbivorous, carnivorous,

and omnivorous groups. Next, the teacher provided a paper containing several animal drawings, and asked the students to cut the paper into pieces of each animal which were then pasted into a table on Student Activity Sheet 1 that had been provided. This activity aims to foster student activity and motivation in learning so that it can create conducive and fun learning. On this occasion, the teacher tried to ask about the cuts of animal images with the aim of re-ascertaining whether they could group the animals based on the type of food.

- Teacher : “Coba sekarang perhatikan tabel dan gambar-gambar yang kalian buat. Apa yang kalian dapatkan? Ada berapa himpunan yang terbentuk?”
- Student1 : “Ada tiga ya bu? Himpunan hewan herbivora, hewan karnivora dan hewan omnivora.”
- Teacher : “Coba sebutkan masing-masing anggotanya.”
- Student : “Untuk yang herbivore ada sapi, kelinci, rusa. Yang karnivora itu ada harimau dan singa. Trus, yang omnivora ada ayam, babi dan monyet.”
- Teacher : “Nah..kira-kira apa hubungan dari tabel yang sudah kalian buat?”
- Student2 : “Hubungannya.....hewan dan jenis makanannya? (in a hesitant tone)”

In this activity, the teacher guides students to identify known things to be able to answer questions. The goal is to ensure and strengthen students' understanding of relationships.

- Teacher : “Dari apa yang sudah kalian buat, apa itu relasi?”
- Student : “Relasi itu adalah hubungan ya bu?”
- Teacher : “Iya, hubungan apa?”
- Student2 : “Hubungan antara himpunan pertama dan himpunan kedua.”

The conversation above shows that through the activity of grouping animals based on the type of food, students are able to understand the concept of relationships even though it can be seen that students are still hesitant to express their opinions about the meaning of relationships. Learning activities also show positive attitudes and responses from students.

At the end of activity 1, the teacher directs students to jointly make conclusions about relationships. The activity continued by declaring the presentation of relationships (arrow diagrams, cartesian diagrams, and sequential pairs) based on the grouping of animals in activity 1. The teacher asks the students to declare relationships based on the results they obtained in activity 1 by asking again the set formed based on the division of the table, namely animals and types of food, which then the results of activity 2 are shown in Figure 5. Based on activity 2, it can be concluded that students are able to express relationships into three forms, namely arrow diagrams, cartesian diagrams, and sequential pairs.

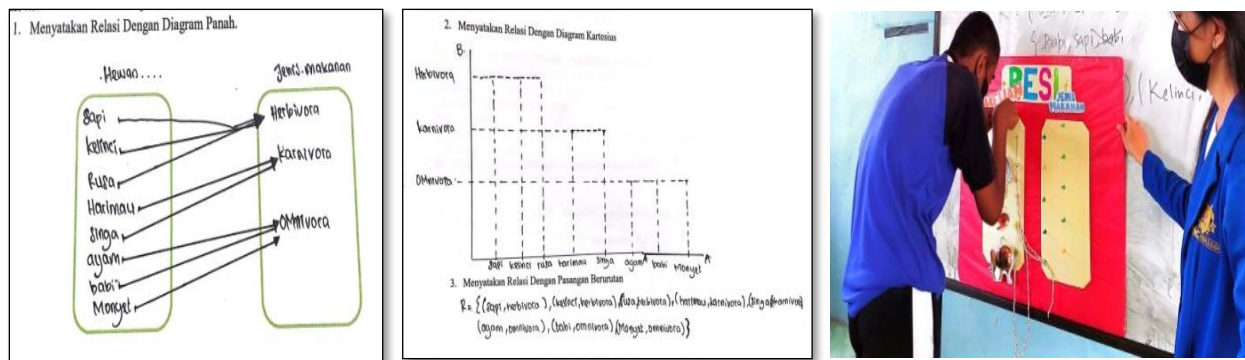


Figure 5. Results of Activity 2

Furthermore, in the activity 3 students were led to understand the domain, codomain and range described in the following conversation.

- Teacher : “Sekarang, coba perhatikan lagi relasi yang terbentuk pada diagram panah, apa himpunan pertamanya dan apa himpunan keduanya?”
- Student : (While looking back at the arrow diagram on activity 2) “Himpunan pertamanya itu hewan dan himpunan keduanya jenis makanan bu.”
- Teacher : “Ya.. Jadi dalam relasi, kita harus mengenal yang namanya daerah asal, daerah kawan, dan daerah hasil. Nah..kira-kira berdasarkan diagram panah tersebut yang manakah yang dikatakan sebagai daerah asal, daerah kawan, dan daerah hasil?”
- Student : (Speechless)
- Teacher : “Ada yang tahu? Coba kita kaji sama-sama kata-nya. Apa arti dari kata daerah?”
- Student3 : “Daerah itu tempat bu”
- Teacher : “Ya..tempat. Yang namanya suatu tempat, contohnya taliwan atau sekolah ini, dipasang pagar kan? Apa artinya dipasang pagar?”
- Student : “Pembatas”
- Teacher : “Betul sekali, ada batas. Nah kalo kita kaji lagi kata asal, kawan, dan hasil?”
- Student : “Kalau asal ya asal bu... kawan itu teman. Terus hasil itu apa yang diperoleh.”
- Teacher : “Berarti apa itu daerah asal, daerah kawan dan daerah hasil? Coba liat kembali diagram panahnya. Kira-kira daerah asal yang mana?”
- Student1 : “Yang himpunan pertama ya bu? (sounding hesitant)”
- Teacher : “Kenapa himpunan pertama?”
- Student1 : “Karena kan asalnya.”
- Teacher : “Kalau daerah kawan?”
- Student1 : “Himpunan kedua karena temannya himpunan pertama”
- Teacher : “Terus daerah hasil yang mana?”
- Student1 : “Daerah hasilnya itu...(was silent for a moment) oh yang ada pasangannya bu.”

Once the student is able to determine the area of origin, the area of friends and the area of outcomes, the teacher asks the students to conclude what they know about the domain,

codomain and range. Then students are led to be able to complete the questions on Student Activity Sheet 1 and present the results of the Student Activity Sheets work they obtained.

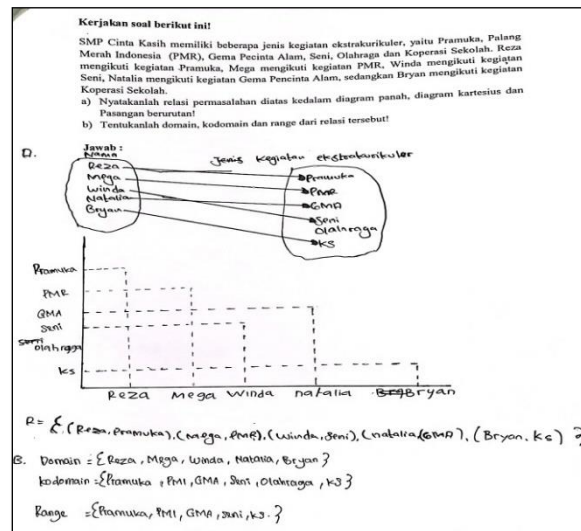


Figure 6. Student Answer Results

Second Meeting: Concept of Function, Characteristics of Function and One-on-one Correspondence

Learning activities at the second meeting were carried out with the aim that students were able to understand the basic concepts and characteristics of a function and state the requirements for one-on-one correspondence. The learning activities carried out are inseparable from the context used at the initial meeting, namely contextual problems that exist around the student environment. In this case, the teacher uses contextual problems, namely: Activity 1 uses the problem of the relationship between names, dates of birth, hobby and preferred subjects, while Activity 2 uses the context of Southeast Asian countries taken from civics subjects.

In activity 1, students are directed to write down their names, dates of birth, hobby and preferred subjects on Student Activity Sheet 2 as a first step in understanding the concept of function. Next, students are asked to state relationships into the form of an arrow diagram. The goal is for students to get a clearer picture of the shape of a function. It can be seen that one mistake made by students when making an arrow diagram of the third relation is that there is a member of the second set that is written repeatedly. And when the teacher asks about it, the students say they don't remember the set material that has been taught before.

Next, the teacher asks the students to observe the relationships formed from the three relationships (arrow diagram). The results of this activity show that there is a process of discussion between students in finding differences or similarities in the three relationships.

However, at first students have difficulty when looking for differences or similarities in the three relationships so that it has an impact on drawing conclusions from the understanding of functions that they understand so that students need more guidance from the teacher. In this case, the teacher directs the students to identify the members of each set as well as the form of the three relationships (arrow diagrams) that are formed and further guides the students to know each pair of members of the set of each relation so that the student's mind can be well directed in understanding the concepts and characteristics of the function.

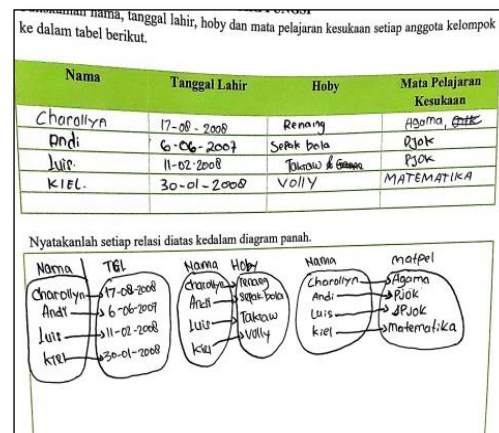


Figure 7. Results of Meeting Activity 2

After understanding the function, the learning activity is continued by understanding co-response one-on-one. The use of the context of Southeast Asian countries is one example that can be used in the RME learning approach. Activity activity 2 begins by asking students to write down and state the relationships of the 5 countries and their capitals into the arrow diagram on the Activity Sheet 2 provided. Next, the teacher handed out a paper containing pictures of the flags of Southeast Asian countries which were then cut by students and directed students to choose flags based on the 5 countries they had written at the beginning of the activity. In this activity, it can be seen that students discuss choosing the country and the capital they know and work together to complete their tasks. The process for understanding one-on-one correspondence is explained based on the following conversations in the classroom.

- Teacher : “Sekarang, perhatikan kedua relasi, yaitu negara dan ibukota, dan negara dan bendera. Jadi, dalam suatu relasi, ada yang dinamakan dengan korespondensi satu-satu. Nah, dari bentuk kedua relasi tersebut, apa yang dapat kalian simpulkan tentang korespondensi satu-satu?”
- Student2 : “Kalo dari bentuknya, anggota himpunan satu sama jumlahnya dengan anggota himpunan dua bu.”
- Teacher : “Ya. Itu satu. Ada yang punya pendapat lain?”
- Student3 : “Hmm... anggotanya punya pasangan masing-masing bu.”

- Teacher : “Anggotanya punya pasangan masing? Maksudnya bagaimana?”
- Student3 : “Kayak Indonesia pasangannya Jakarta, Malaysia pasangannya Kuala Lumpur, Filipina pasangannya Manila. Semuanya ada bu.”
- Teacher : “Ya, betulnya. Jadi, anggota kedua himpunan baik itu pada relasi satu dan relasi dua itu masing-masing memiliki pasangan yang tepat satu. Berarti apa itu korespondensi satu-satu?”
- Student : “Korespondensi satu-satu itu, anggota himpunan pertama punya tepat satu pasangan dengan anggota himpunan kedua.”

Based on these conversations, it can be concluded that students are able to understand the terms of a relationship can be said to be a one-on-one correspondence.

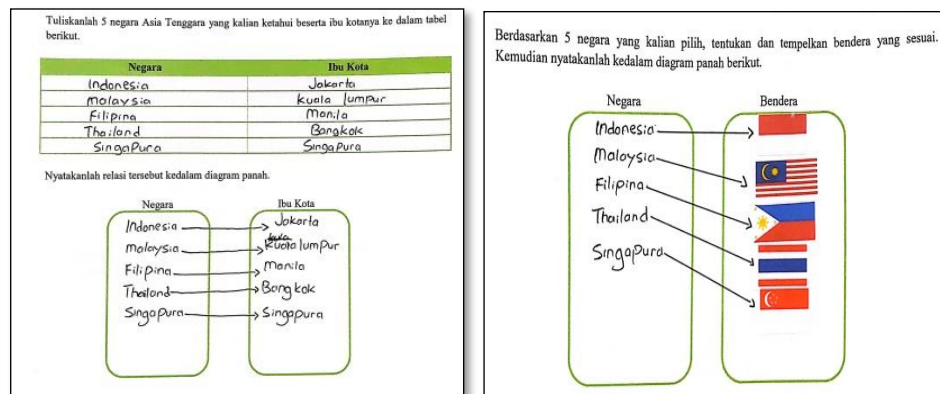


Figure 8. One-on-one Correspondence

Third Meeting: Notation and Value of Functions

In the third meeting activity, the teacher presents problems that are in accordance with the learning objectives to be carried out, namely understanding writing and knowing how to determine the value of functions. The existing problems presented in the form of modified questions have two ways of solving, namely (1) by using a table assisted by Rp.50,000 toy money, and (2) using function formulas. This is done to make it easier for students to understand the use of function notation and bridge the student's thought process from contextual mathematics to abstract mathematics. As the first step of the learning activity, the teacher asks students to read instructions and materials regarding writing function notation that have been listed on student activity sheet 3. This activity is carried out to provide information about writing function notation so that it can anticipate errors when students solve problems when using function formulas.

The process of solving problem problems is carried out using a toy money tool of Rp.50,000 with the aim that students can understand the process of solving existing problems. The teacher gave several rp.50,000 bills to the students and asked the students to demonstrate saving activities according to what was written on the question. When students solve

problems in the problem using toy money, the teacher continues learning activities using the function formula, whose conversation is described as follows.

- Teacher : “Berdasarkan soal yang ada, berarti disini kita harus memisalkan x dan $f(x)$ dengan apa?”
 Student : “ x itu lama menabung, kalo $f(x)$ jumlah tabungannya bu.”
 Teacher : “Terus, bagaimana dengan rumus fungsinya?”
 Student : “(After discussing for a few seconds), rumus fungsinya itu 50.000 kali x bu.”

Based on these conversations, it can be seen that students are able to process their thinking in finding function formulas based on existing problems. Students are also able to solve problems based on the results of the discussions they have. This shows that the use of the context of savings questions using the help of Rp.50,000 toy money can open students' insights to solve problem solving problems related to function.

Perhatikan Permasalahan Berikut.

Pada Januari, Fiki mulai menabung di Bank dengan tabungan awal sebesar Rp.50.000. Jika setiap bulannya Fiki rutin menabung dengan jumlah yang sama dengan tabungan awal, berpakah jumlah tabungan Fiki pada bulan Mei?

Jawab :

Dik : Tabungan awal bulan Januari sebesar Rp.50.000.
 Setiap bulan Fiki rutin menabung uang sebesar Rp.50.000

Dit : Jumlah tabungan Fiki pada bulan Mei?

Penye :

Untuk dapat menemukan jumlah tabungan Fiki pada bulan Mei, maka lengkapi tabel berikut ini.

Bulan	Jumlah setoran	Jumlah Keseluruhan Tabungan
Januari	Rp. 50.000	Rp.50.000
Februari	Rp. 50.000	Rp. 100.000
Maret	Rp. 50.000	Rp. 150.000
April	Rp. 50.000	Rp. 200.000
Mei	Rp. 50.000	Rp. 250.000

Berdasarkan tabel tersebut, diperoleh jumlah tabungan Fiki pada bulan Mei adalah Rp.250.000

Dalam matematika, kita bisa menyelesaikan permasalahan tersebut dengan rumus fungsi dengan memisalkan.

Mis. x = lama menabung
 $f(x)$ = Jumlah tabungan pada bulan ke- (x)

Oleh karena, tabungan Fiki bertambah sebanyak Rp.50.000 setiap bulan maka :

$$f(x) = \dots 50.000 \times x \dots$$

Perhatikan bahwa,

Januari $\rightarrow$ bulan ke-1. Maka $x = 1$.
 Februari $\rightarrow$ bulan ke-2. Maka $x = 2$.
 Maret $\rightarrow$ bulan ke-3. Maka $x = 3$.
 April $\rightarrow$ bulan ke-4. Maka $x = 4$.
 Mei $\rightarrow$ bulan ke-5. Maka $x = 5$.

Untuk menentukan jumlah tabungan Fiki pada bulan Mei, Maka kita substitusikan $x = 5$ ke $f(x)$, maka

$$f(x) = \dots 50.000 \times x \dots$$

$$f(5) = \dots 50.000 \times 5 \dots = \text{Rp. } 250.000$$

Jadi, jumlah tabungan Fiki pada bulan Mei adalah Rp. 250.000.

Figure 9. Problem Solving Answer Results

Next, the teacher asks the students to solve the problem-solving problem in Student Activity Sheet 3. The following are the results of the answers done by students:

Soal

Sebuah bak mandi berisi air sebanyak 5 liter. Kemudian Rafi menyalakan keran air yang dimana air di dalam bak akan bertambah 2 liter per menit. Berapakah banyak air di bak mandi pada menit ke-6?

Jawab :

Dik : Air di bak sebanyak 5 liter.
 Kemudian bak akan bertambah sebanyak 2 liter per-menit.

Dit : Berapa banyak air di bak mandi pada menit ke-6?

Penye :

Mis. x = mengisi waktu waktu mengisi air bak mandi.
 $f(x)$ = jumlah air pada menit ke- x

Oleh karena, air bak akan bertambah sebanyak 2 liter setiap menitnya, maka :

$$f(x) = \dots 5 \dots + 2 \times (x)$$

Perhatikan

Banyak air sebelum keran dinyalakan = 5 liter
 Tambahan air menit pertama ($x = 1$) = 2. liter
 Tambahan air menit kedua ($x = 2$) = 2. liter
 Tambahan air menit ketiga ($x = 3$) = 2. liter
 Tambahan air menit keempat ($x = 4$) = 2. liter
 Tambahan air menit kelima ($x = 5$) = 2. liter
 Tambahan air menit keenam ($x = 6$) = 2. liter

Untuk menentukan banyak air pada menit ke-6, Maka kita substitusikan $x = 6$ ke $f(x)$, sehingga diperoleh

$$f(x) = \dots 5 \dots + 2 \times (x)$$

$$f(6) = \dots 5 \dots + 2 \times 6 = 5 + 12 = 17$$

Jadi, banyaknya air pada menit ke-6 adalah 17 liter.

Figure 10. Problem Solving Problem Answer Results

DISCUSSION

Based on the results of the second trial, it can be concluded that the revised learning trajectory hypothesis is in accordance with the expected learning objectives. The revised learning activities on the learning trajectory hypothesis are as follows (1) there is a review activity of the concept of the set by mentioning as many examples of the set as possible, accompanied by the writing of the set, and (2) Selection of relationship examples that will be used in the activity of understanding the concept of relationships.

The overall designed learning trajectory hypothesis does not indicate any obstacles large enough to be faced. Students are able to understand and apply the concept of relationships and functions in solving the given problem problems. Based on the results of the learning trajectory in finding the concept of relationships, learning activities are directed at the process of grouping animals based on the type of food. This shows that the use of animal grouping problems becomes part of the characteristics of Realistic Mathematics Learning, that is, the use of context as well as the existence of a link between the context and the material used. Furthermore, the problems used are modeled through the use of "RESI" to provide a clearer understanding of the concept for students. In addition, the activity of cutting and pasting animal images shows the contribution of students in finding the concept of relationships. The entire process of learning activities is inseparable from the interaction between teachers and students and interaction between students.

On the learning trajectory in understanding the concept of function, the use of context used is directly related to his life, namely creating relationships based on names, dates of birth, hobbies, and favorite subjects. Furthermore, students are asked to state the relationship into the arrow diagram and investigate any differences and similarities to the relationships formed. This is a process of using a mathematical model in which students are directed to understand the characteristics of the function based on the arrow diagram of the relationships they find. In addition, this activity involves the interaction and contribution of students in finding the concept of function. The same thing happens in the process of understanding the concept of one-on-one correspondence which in its activity uses the problems of Southeast Asia.

In the learning track of meeting three, the use of the context used is a real problem which involves the use of toy money Rp.50,000. This activity shows a process of interaction between students and students with teachers. Students are then asked to demonstrate saving activities according to the questions asked. This activity is a characteristic of the contribution and construction of students from Realistic Mathematics Learning. In addition, in the process

of solving the problem, a mathematical modeling process is obtained, namely by providing x as the length of savings and $f(x)$ as the amount of savings. The use of toy money of Rp.50,000 helps students to understand the concept of the problem and find steps to solve the problem.

CONCLUSION

Based on the results of the research above, it can be concluded that the learning trajectory in the Relation and Function material with a RME approach designed to improve mathematical problem solving ability can be applied in the learning process. The resulting learning path is in the form of a flow of activities and activities carried out by students in class in order to achieve the learning goals that have been previously set. Learning activities are carried out in three meetings, namely 1) the first meeting aims to enable students to know the concept and presentation of relationships and determine domains, codomains and ranges, 2) the second meeting aims to enable students to understand the concepts of relationships and correspondence one-on-one, and 3) the third meeting aims to enable students to know notation and find the value of functions.

Based on these results, the learning trajectory designed to improve problem-solving ability is in accordance with the characteristics of Realistic Mathematics Learning, which involves the use of context, mathematical modeling, student contributions, interaction, and the existence of a relationship between context and material. The learning process that uses contextual problems encourages students to think actively so that students get a real understanding of the concepts of relationships and functions. In addition, with the help of "RESI", students are able to find a clearer picture of the concept of relationships and functions. The process of learning activities that occurs takes place positively because it involves the active participation of students so as to increase student motivation in learning. The interaction between teachers and students and students with students also creates a conducive and fun learning atmosphere. Thus, the learning trajectory obtained is expected to be able to overcome problems, especially in the material of relationships and functions in class VIII.

RECOMMENDATIONS

Based on the result of the research above, the author suggests several things, namely 1) to all mathematics teachers to be able to apply the learning trajectory that has been developed in Realistic Mathematics Education in the classroom to learning Relations and Functions, 2)

students to be able to improve problem solving skills by learning using problems in the surrounding environment, and 3) researchers are next to be able to develop a learning trajectory hypothesis on Realistic Mathematics Education for other mathematics subject matter.

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